Statistical Reasoning
Normal Distribution – Z-Scores

Name: ________________ Date: ________________ Class: ________________

Standardizing the Normal Distribution: The Z-Score

In a normal distribution, all data can be analyzed by using **z-scores**.

sample mean → $\bar{X}$; population mean → μ

Z-score formula: $\dfrac{X-\bar{X}}{\sigma} = \dfrac{X-\mu}{\sigma}$

z → The # of standard deviations that a value is located away from the mean is also called the "Standard Score", "sigma" or "z-score".

x → A particular data value

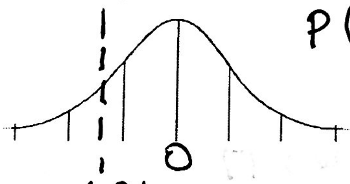
μ → Mean (According to the CLT, given enough samples, a sample mean approximately approaches the population mean)

or $\bar{X}$

σ → Standard Deviation (According to the CLT, given enough samples, a sample standard deviation approximately approaches the population standard deviation)

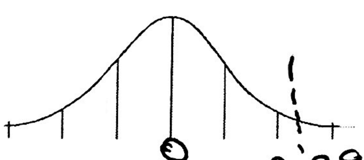
Example: Use the Z-Table to find the proportion of observations. Sketch a standard Normal curve and shade the area under the curve that is the answer to the question.

(a)z is less than -1.26



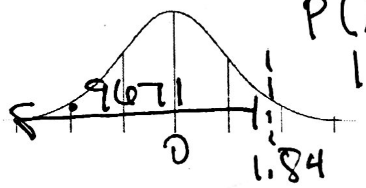
$P(X < -1.26)$
$= .1038$
$\boxed{10.38\%}$

(b)z is less than 2.09



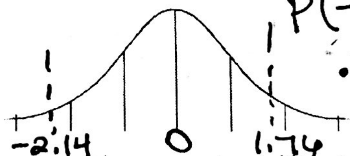
$P(X < 2.09)$
$= .9817$
$\boxed{98.17\%}$

(c) z is greater than 1.84



$P(X > 1.84)$
$1 - .9671$
$.0329$
$\boxed{3.29\%}$

(d) z is between -2.14 and 1.76



$P(-2.14 < x < 1.76)$
$.9608 - .0162$
$.9446$
$\boxed{94.46\%}$

Example: If the mean test score in a class is 62 (Mean) with a standard distribution of 8. Mr. H scored a 79, what percent di he score higher than?

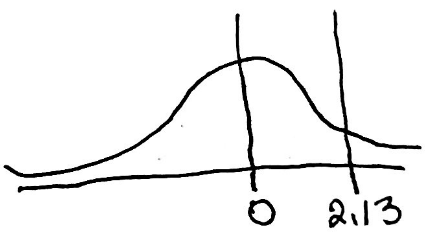
3. Convert to a probability. Use the z-table to find the z-score and the accompanying probability

1. Identify the probability statement: $P(X < 79) = .9834 = \boxed{98.34\%}$

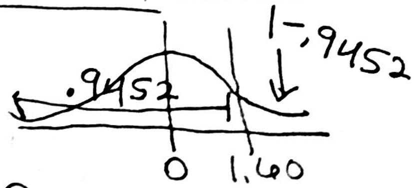
2. Calculate:

$$z-score = \frac{79-62}{8} = \frac{17}{8} = 2.13$$

79 is 2.13 standard deviations more than the mean of 62.



Example: You are the director of transportation safety for the state of Georgia. You are concerned because the average highway speed of all trucks may exceed the 60 mph speed limit. Assuming that the population mean is 60 mph and population standard deviation is 12.5 mph, find the probability of an average speed more than or equal to 80 mph.



1. Identify the probability statement: $P(X \geq 80)$
2. Calculate:

$$z - score = \frac{80-60}{12.5} = 1.60$$

80 mph is 1.40 standard deviations more than the mean of 60.

3. Convert to a probability. Use the z-table to find the z-score and the accompanying probability

$1 - .9452 = .0548 = \boxed{5.48\%}$

Example: You are touring the country playing music from city to city. On average, the band receives $\bar{x}$ $1,257 per night (from merchandise and revenue from tickets sales). The amount of income has ~~normal distribution~~ σ st. dev of $272. It been a "pretty good"concert if the band sells between $1,000 and $1,500 for the night; what percent of the shows are "pretty good."

4. Identify the probability statements: $P(1000 < X \leq 1500)$
5. Calculate:

$$z - score = \frac{1000-1257}{272} = -0.94 \quad z - score = \frac{1500-1257}{272} = 0.89$$

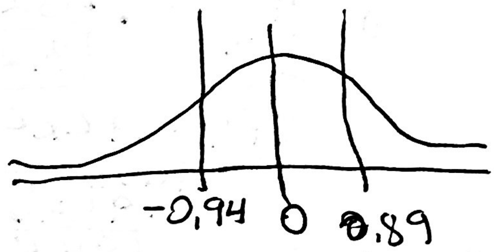
$1,000 is -0.94 standard deviations more than the mean of $1,257.

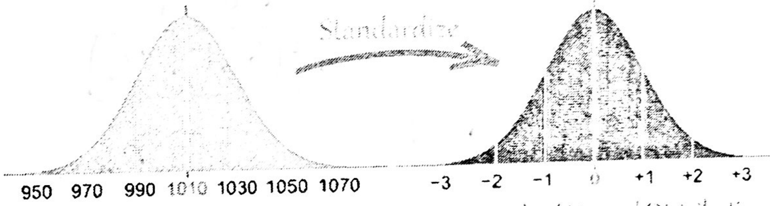
$1,500 is 0.89 standard deviations more than the mean of $1,257.

6. Convert to a probability. Use the z-table to find the z-score and the accompanying probability

$.8133 - .1736 = .6397$

$\boxed{63.97\%}$





A Normal Distribution The Standard Normal Distribution

Why it matters

Standardizing a normal distribution is useful because is enables values from different normal distributions to be easily compared values that can't be easily calculated with the Empirical rule.

When used in conjunction with a z score table, z-scores can be converted into probabilities and percentages.