Statistical Reasoning
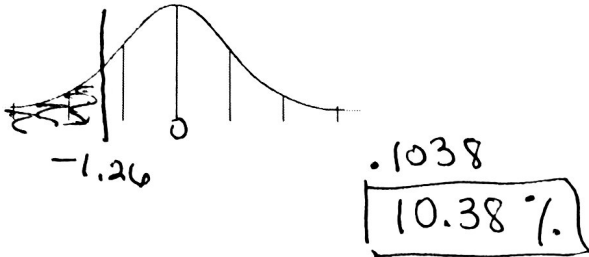**Normal Distribution – Z-Scores**

Name: _____Key_____ Date: _____ Class: _____
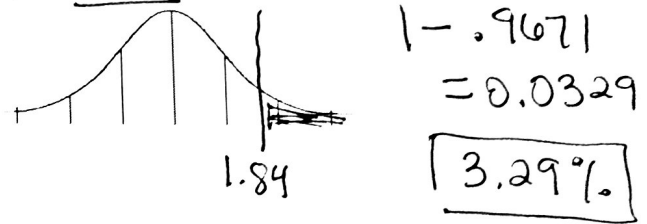
①

Normal Distributions Practice

1. Use the Z-Table to <u>find the proportion</u> of observations from a standard Normal distribution that satisfies each of the following statements. In each case, <u>sketch a standard Normal curve</u> and <u>shade the area</u> under the curve that is the answer to the question.
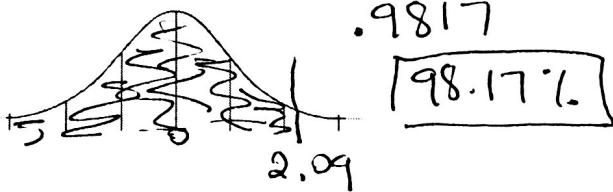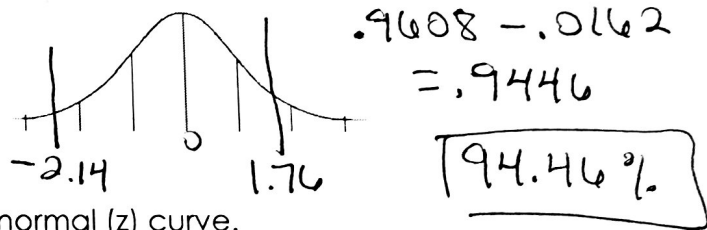
(a) z is less than -1.26

−1.26

.1038

$\boxed{10.38\ \%}$

(c) z is greater than 1.84

$1-.9671$
$=0.0329$

1.84

$\boxed{3.29\ \%}$

(b) z is less than 2.09

2.09

.9817

$\boxed{98.17\ \%}$

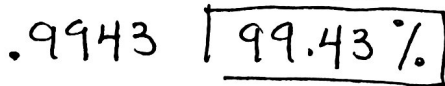(d) z is between -2.14 and 1.76

−2.14   1.76

$.9608-.0162$
$=.9446$

$\boxed{94.46\ \%}$
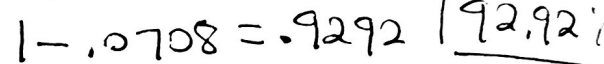
2. Determine the following areas under the standard normal (z) curve.

(a) The area under the z curve to the <u>left</u> of 2.53.

.9943 $\boxed{99.43\ \%}$
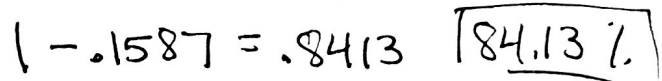
(d) The area under the z curve to the <u>right</u> of −1.47

$1-.0708=.9292$ $\boxed{92.92\ }$

(b) The area under the z curve to the <u>left</u> of −1.33

.0918 $\boxed{9.18\ \%}$

(e) The area under the z curve <u>right</u> of −1

$1-.1587=.8413$ $\boxed{84.13\ \%}$

(c) The area under the z curve <u>between</u> 0.76 and 1.67

$.9525-.7764$
$.1761$

$\boxed{17.61\ \%}$
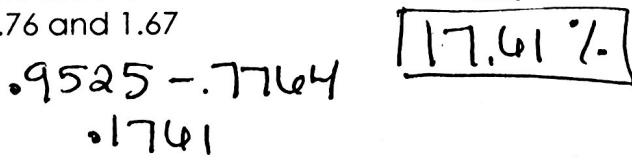
(f) The area under the z curve <u>between</u> -0.61 and 0.34

$.6331-.2709$
$.3622$

$\boxed{36.22\ \%}$
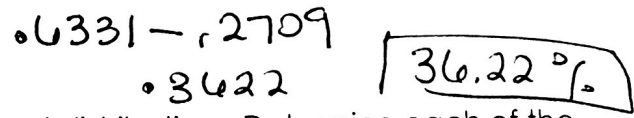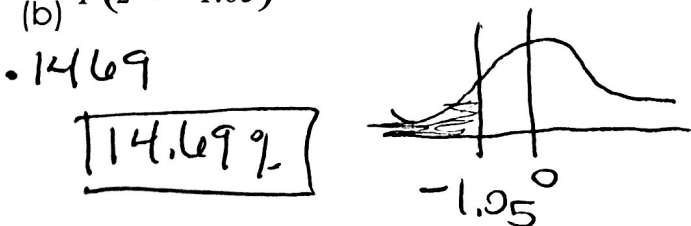
3. Let z denote a random variable having a standard normal distribution. Determine each of the following probabilities.

(a) $P(z < 1.28)$

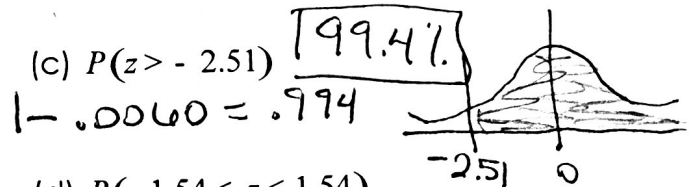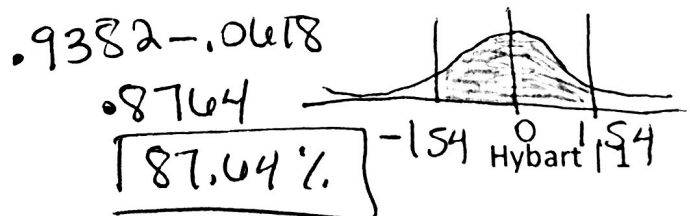.8997 $\boxed{89.97\%}$

0 1.28

(b) $P(z < -1.05)$

.1469

$\boxed{14.69\ \%}$

−1.05 0

(c) $P(z > -2.51)$ $\boxed{99.4\%}$

$1-.0060=.994$

−2.51 0

(d) $P(-1.54 < z < 1.54)$

$.9382-.0618$
$.8764$

$\boxed{87.64\ \%}$

−1.54 0 1.54
Hybart
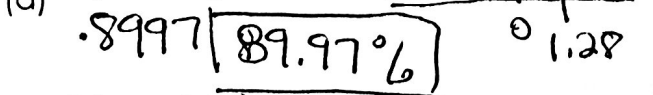
4. A gasoline tank for a certain model car is designed to hold 12 gallons of gas. Suppose that the actual capacity of the gas tank in cars of this type is well approximated by a normal distribution with mean 12.0 gallons and standard deviation 0.2 gallons. What is the probability that a randomly selected car of this model will have a gas tank that holds at most 11.7 gallons? $P(x \leq 11.7)$

$z = \dfrac{11.7 - 12}{0.2} = -1.5$

.0668 $\boxed{6.68\%}$

−1.5 0
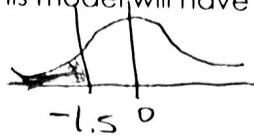
5. The owners of the Burger Emporium are looking for new supplier of onions for their famous hamburgers. It is important that the onion slice be roughly the same diameter as the hamburger patty. After careful analysis, they determine that they can only use onions with diameters less than 10 cm. Company A provides onions with diameters that are approximately normally distributed with mean 10.3 cm and standard deviation of 1.2 cm. Company B provides onions with diameters that are approximately normally distributed with mean 10.6 cm and standard deviation of 0.9 cm. Which company provides the higher percentage of usable onions? Justify your choice with an appropriate statistical argument. $P(x < 10)$

Company A (Better)

$z = \dfrac{10 - 10.3}{1.2} = -0.25$ .4013 $\boxed{40.13\%}$

Company B

$z = \dfrac{10 - 10.6}{0.9} = -.67$ .2514   25.14 %

6. The overall batting average of all major-league baseball hitters in recent years (1970 – 2008) has followed a roughly normal distribution with a mean of .261 and standard deviation .035.

(a) About what percent of hitters bat .226 or lower? (Sketch a normal curve to illustrate and use the 68-95-99.7 rule.)

.15 + 2.35 + 13.5

$= \boxed{16\%}$

0 .226 .261
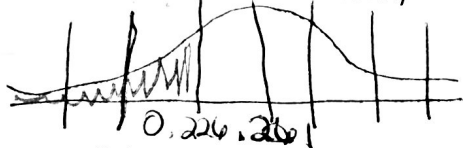
(b) What percent of hitters bat .331 or higher? (Sketch a normal curve to illustrate and use the 68-95-99.7 rule.)
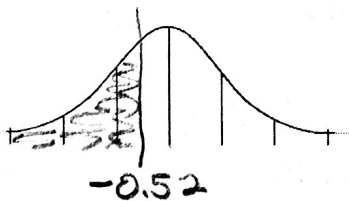
34 13.5 2.35 .15

$\boxed{2.5\%}$

.261 .296 .331

7. GOING BACKWARDS: Use the Z-Table A to find the **closest** value to satisfying the condition. In each case sketch a standard Normal curve with your value of z marked on the axis.

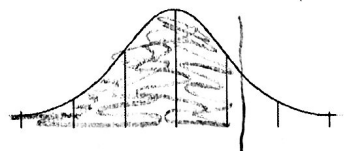(a) The 30th percentile of the standard normal distribution   .3015

$z = -0.52$

−0.52

(b) 7% of all observations are greater than z

93% to left
1.48 = z

1.48

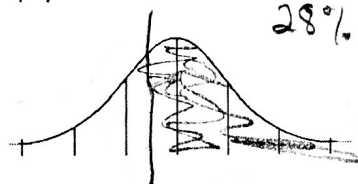(c) The 88th percentile of the standard Normal distribution $1.17 = z$

1.17
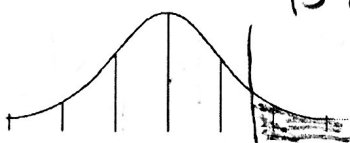
(d) 72% of all observations are greater than z

28% to left
$z = -0.58$

−0.58

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