Warm Up

Divide using polynomial long division.

$n+3=0 \quad n=-3$

1) $(9n^3 + 28n^2 + 11n + 16) \div (n + 3)$

$$
\begin{array}{r}
9n^2 + n + 8 \\
n+3 \,\big)\, 9n^3 + 28n^2 + 11n + 16 \\
-\;9n^3 + 27n^2 \qquad\qquad \\
\hline
n^2 + 11n + 16 \\
-\;n^2 + 3n \qquad \\
\hline
8n + 16 \\
-\;8n + 24 \\
\hline
-8
\end{array}
$$

$$\boxed{9n^2 + n + 8 + \frac{-8}{n+3}}$$

Synthetic Division

$$
\begin{array}{r|rrrr}
-3 & 9 & 28 & 11 & 16 \\
 & \downarrow & -27 & -3 & -24 \\
\hline
 & 9x^2 & 1x & 8 & -8
\end{array}
$$

Rem

$$\boxed{9x^2 + x + 8 + \frac{-8}{n+3}}$$

① Bring Down
② mult by box
③ Add
④ Repeat 2-4

Factor Theorem

A polynomial $f(x)$ has a factor $(x - k)$ if and only if $f(k) = 0$.

Example

Given Function with $f(k) = 0$.

$f(x) = 2x^3 + 11x^2 + 18x + 9$
given $f(-3) = 0$
-3 is a solution

Use synthetic division to find the other factors.

$$
\begin{array}{r|rrrr}
-3 & 2 & 11 & 18 & 9 \\
 & \downarrow & -6 & -15 & -9 \\
\hline
 & 2x^2 & 5x & 3 & \mid 0
\end{array}
$$

Interpret the 3rd row as a quadratic expression.

$2x^2 + 5x + 3$

Factor the resulting polynomial.

$2x^2 + 5x + 3$ $\quad$ 6: 2, 3; 5

$x^2 + 5x + 6$

$(x + \frac{2}{2})(x + \frac{3}{2})$

$(x+1)(2x+3)$

$x + 1 = 0 \quad 2x + 3 = 0$

$x = -1 \quad x = -3/2$

Factors:

Write your polynomial in factored form.

$(x+3)(x+1)(2x+3)$

Zeros: $-3, -1, -3/2$

Your Turn

$f(x) = 2x^3 - 3x^2 - 32x - 15$
given $f(5) = 0$

5	2	-3	-32	-15
	↓	10	35	15
	2	7	3	0

$2x^2 + 7x + 3$

$x^2 + 7x + 6$

$\left(x + \frac{6}{2}\right)\left(x + \frac{1}{2}\right)$

6, 1, 7, 6

Factors $(x+3)(2x+1)(x-5)$

Zeros $x = -3,\ x = -\frac{1}{2},\ x = 5$

Find the quotient and remainder for the following polynomial division problems. Indicate whether the divisor is a factor. If "yes", then identify a zero of the polynomial.

EX 1: $(x^3 + 7x^2 + 3x + 1) \div (x + 2)$	Your turn: $(x^3 + 8x^2 + 8x - 32) \div (x + 4)$
$\begin{array}{r\|rrrr} -2 & 1 & 7 & 3 & 1 \\ & \downarrow & -2 & -10 & 14 \\ \hline & 1 & 5 & -7 & \vert\, 15 \end{array}$ No, $(x+2)$ is Not a factor.	$\begin{array}{r\|rrrr} -4 & 1 & 8 & 8 & -32 \\ & & & & \\ \hline & 1 & 4 & -8 & \vert\, 0 \end{array}$ Yes $(x+4)$ is a factor -4 is a solution

EX 2: $P(x) = 3x^3 + 8x^2 - 3x - 8$ given that $(x - 1)$ is a factor.

$$\begin{array}{r|rrrr} 1 & 3 & 8 & -3 & -8 \\ & \downarrow & 3 & 11 & 8 \\ \hline & 3 & 11 & 8 & \big| \ 0 \end{array}$$

$3x^2 + 11x + 8$

$x^2 + 11x + 24$

24

8 3

11

$\frac{x+8}{3} = 3x + 8$

$\frac{x+3}{3} = x + 1$

Factors

$(3x+8)(x+1)(x-1)$

$3x + 8 = 0$
$-8 \quad -8$

$\frac{3x}{3} = \frac{-8}{3}$

Zeros

$x = -8/3 \quad x = -1 \quad x = 1$

EX 3: $P(x) = x^3 - 12x^2 + 12x + 80$ given that $(x - 10)$ is a factor

EX 4: $P(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$ given that P(2) = 0 and x= -3

$(x-2)$

$$\begin{array}{r|rrrrr} 2 & 2 & 7 & -4 & -27 & -18 \\ & \downarrow & 4 & 22 & 36 & 18 \\ \hline & 2 & 11 & 18 & 9 & 0 \end{array}$$

$2x^3 + 11x^2 + 18x + 9 \mid 0$

$$\begin{array}{r|rrrr} -3 & 2 & 11 & 18 & 9 \\ & \downarrow & -6 & -15 & -9 \\ \hline & 2 & 5 & 3 & 0 \end{array}$$

$2x^2 + 5x + 3 \mid 0$

$2x^2 + 5x + 3$

6, 3, 2, 5

$x^2 + 5x + 6$

$(x + \frac{3}{2})(x + \frac{2}{2})$

Factors

$(2x+3)(x+1)(x-2)(x+3)$

Zeros $x = 2, -3, -1, -\frac{3}{2}$

Zeros 2, -3,