Statistical Reasoning
Hypothesis Tests

Name: Date: Class:

Inference with a Z: Central Limit Theorem in context

1. A bottling company uses a filling machine to fill plastic bottles with a popular cola. The bottles are supposed to contain 300 milliliters (ml). The company claims that they overfill their bottles because their contents vary according to a normal distribution with mean μ = 303 ml and standard deviation σ = 7 ml.

   a. State the null and alternative hypothesis.

   H₀ (the claim): μ = 303 ml **Hₐ:** μ > 303ml

   b. What is the probability that an individual bottle contains less than 299 ml?

   $z = \frac{299-303}{7} = -0.57$ .2843 $\boxed{28.43\%}$

   c. Suppose a sample of 10 bottles were taken off the production line. What is the probability that the sample mean contents of the 10 bottles is less than 299 ml?

   $z = \frac{299-303}{\frac{7}{\sqrt{10}}}$ 0.035 (normcdf) $\boxed{3.5\%}$

   If the company's claim of 303 ml is true, you should expect to find that a random sample of 10 bottles would have a mean volume of 299 ml (4 ml less than claimed population mean), 3.5 % of the time.

   d. Is there evidence that the claim is true?

______ = ______
______ = ______

   e. What reasons might the bottling company have a different sample mean from the claimed population mean?

2. In a study done on the career expectancy of 500 teachers in the state of Georgia, the mean age at burn out was 32 years old with a standard deviation of 5.3 years.

   a. State the null and alternative hypothesis.

   H₀ (the claim): μage = 32 years **Hₐ:** μage ≠ 32 years

   b. What is the probability that an individual will be less than 30 years old?

   $z = \frac{30-32}{5.3} = -0.38$ .3520

   (or cdf) $\boxed{35.20\%}$

c. If a random sample of 50 teachers were selected, what would be the probability tha
the sample mean if less than 30 years old?

$z = \frac{30-32}{\frac{5.3}{\sqrt{50}}} = -2.67$ $.0038$ $\boxed{.38\%}$

(or cdf, $\sigma = 5.3/\sqrt{50}$)

If the claim of teacher's burnout age is true, you should expect to find that .38 %, o
random sample of 50 teachers, are burnt out by the mean age of 30.

d. Is there evidence that the claim is true?

e. Why might the sample have provided some evidence about teachers burning out before the claimed mean age of 32?

3. When Hybart Hardware's equipment is working well, it produces steel bolts with a breaking strength that is normally distributed with a mean of 520 lbs. and a standard deviation of 5 lbs. A random sample of 25 bolts is selected twice a day and the bolts stressed to their breaking point to determine their strength. The latest sample had a mean breaking strengt 518 pounds.

a. State the null and alternative hypothesis.

Ho (the claim): $\mu_{strength} = 520$ lb **Ha:** $\mu_{strength} \neq 520$ lb

b. What is the probability that the sample mean of the 25 bolts is less than 518 lbs?

$z = \frac{518-520}{\frac{5}{\sqrt{25}}} =$ 0.0228 $\boxed{2.28\%}$

If the Hybart Hardware's mean bolt breaking point is true, you should expect to find tha random sample of 25 bolts would have a mean breaking point of 518 lbs, 2.28 % of time.

c. Is there evidence that the claim is true?

4. The mean area of the several thousand apartments in a new development is advertised to b 1250 square feet with a standard deviation of 100 sq feet. A tenant group thinks that the apartments are smaller than advertised. They hire an engineer to measure a sample of 10 apartments to test their suspicion.

a. State the null and alternative hypothesis.

Ho (the claim): $\mu = 1250$ ft^2 **Ha:** $\mu < 1250$ ft^2

b. What is the probability that the sample mean of the 10 apartments are less than 1200 sq. Feet?

$z = \frac{1200-1250}{\frac{100}{\sqrt{10}}}$ 0.0569 $\boxed{5.7\%}$

If the developer's average of 1250 sq. ft claim is true, you should expect to find that 5.7 %, of a random sample of 10 apartments, have less than 1200 sq. feet

c. Is there evidence that the claim is true?

5. The nicotine content in milligrams (mg) in E - cigarettes of a certain brand is normally distributed with a standard deviation of 0.1 mg. The brand advertises that the mean nicotine content of its cigarettes is 1.5 mg, but measurements on a random sample of 100 E-cigarettes of this brand gave a mean of 1.53 mg.

a. State the null and alternative hypothesis.

H0 (the claim): $\mu = 1.5$ mg **Ha:** $\mu \neq 1.5$ mg

b. What is the probability that the sample mean of the 100 e-cigarettes have more than 1.53 mg of nicotine?

$z = \frac{1.53 - 1.5}{0.1/\sqrt{100}}$ $1 - .9987 = 0.0013$ $\boxed{.13\%}$

If the E-cigarette brand's claim about the mean nicotine level is true, you should expect to find that a random sample of 100 e-cigarettes would have a mean nicotine level more than 1.53 mg, .13 % of the time.

c. Is there evidence that the claim is true?

6. An engineer designs an improved light bulb. The previous design claims to have an average lifetime of 1200 hours with a standard deviation of 50 hours. A random sample of 2000 new bulbs is found to have a mean lifetime of 1201 hours.

a. State the null and alternative hypothesis.

H0 (the claim): $\mu = 1200$ hours **Ha:** $\mu \neq 1200$ hours

b. What is the probability that the sample mean of the 2000 bulbs have a lifetime of 1201 hours?

$z = \frac{1201 - 1200}{50/\sqrt{2000}}$ .8144

If the engineer's claim of a mean 1200 lifetime hours is true, you should expect to find that a random sample of 2000 new bulbs to have a mean of 1201 lifetime hours, 81.44 % of the time.

c. Is there evidence that the claim is true?

7. The North Cobb High School (NCHS) published a report in 2017 which indicated that students had broken an average of 3 cellphone screens in their lifetime with a standard deviation of 2.3 screens. A student sampled 10 of their peers about their shattered cell phone screens and found their mean to be 4.

a. State the null and alternative hypothesis.

H0 (the claim): $\mu = 3$ screens **Ha:** $\mu \neq 3$

b. What is the probability that the sample mean of the 10 students have shattered more than 4 cell phone screen? $z = \frac{4-3}{2.3/\sqrt{10}}$ $1 - .9154 = 0.0846$

If the publisher's claim about students breaking an average of 3 screens in true, you should expect to find that a random sample of 10 student's to have broken more than 4 screens, 8.5 % of the time.

c. Is there evidence that the claim is true?

8. A particular statistics teacher believes his students to be much more intelligent than the typical statistics student. On a national standardized test the average score was a 75 with a standard deviation of 18. The teacher randomly selected 15 of his students to take the test. When the scores were returned he found that the student had scored an average of 80.

a. State the null and alternative hypothesis.

H₀ (the claim): $\mu = 75$ **Hₐ:** $\mu > 75$

b. What is the probability that the sample mean of the 15 students was 80?

$z = \frac{80-75}{18/\sqrt{15}}$ .95899

c. Write a sentence that explains your answer from part b.

d. Is there evidence that the claim is true?

9. Adults can read four paragraphs of text in the common Times New Roman font in an average time of 22 seconds and standard deviation of 5 seconds. Researchers asked a random sample of 24 adults to read this text in the ornate font named Gigi and found that their four paragraph reading time to have a mean of 23 seconds.

a. State the null and alternative hypothesis.

H₀ (the claim): $\mu = 22$ seconds **Hₐ:** $\mu \neq 22$ seconds

b. What is the probability that the sample mean of the 24 adults read 4 paragraphs slower than 23 seconds?

$z = \frac{23-22}{5/\sqrt{24}}$ 0.9928

c. Write a sentence that explains your answer from part b.

d. Is there evidence that the claim is true?

10. Consider all the previous scenarios. In your opinion, what would you consider to be a significant percentage to refute or support the claims?

_____ % →
_____ % →
_____ % →