

**Statistical Reasoning  
Normal Distribution - Z-Scores**

key skip #5,7 (3)

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Class: \_\_\_\_\_

Forward/Backward Normal Distribution Problem

1. Use the z-table to find the value z of a standard normal variable that satisfies each of the following conditions. Sketch a standard normal curve with your value of z marked. The Normal distribution has a mean of 10 and a standard deviation of 1.5.

(a) The point where 70% of the observations falling below the mean.

$z = 0.52$



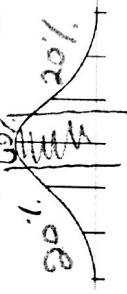
(b) The point where 40% of the observations falling above the mean.

$z = 0.25$



(c) The points where the middle 60% lie around the mean

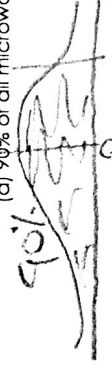
40% outside  $z$ -score - 20%  $z$ -score = -0.84  
 20%  $z$ -score = 0.84



2. Kitchen appliances don't last forever. The lifespan of all microwave ovens is Normally distributed with a mean of 9 years and a standard deviation of 2.5 years.

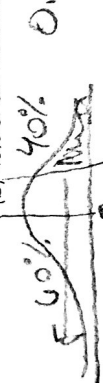
(a) 90% of all microwaves last less than how many years?

$1.28 = \frac{x-9}{2.5} \quad | \quad x = 12.2 \text{ years}$



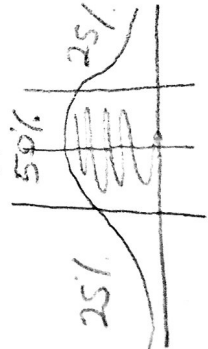
(b) 40% of all microwaves last longer than how many years?

$0.25 = \frac{x-9}{2.5} \quad | \quad x = 9.63 \text{ years}$



(c) The middle 50% of microwaves last between how many years?

$-0.67 = \frac{x-9}{2.5} \quad | \quad 0.67 = \frac{x-9}{2.5}$



$x = 7.33 \text{ to } x = 10.67 \text{ years}$

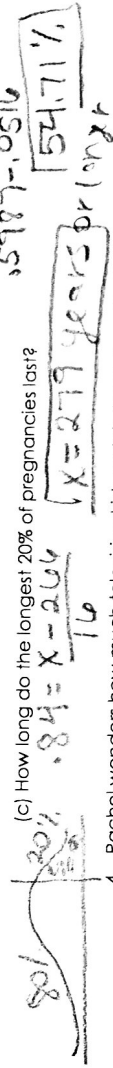
3. The length of human pregnancies from conception to birth varies according to a distribution that is approximately normal with mean 266 days and standard deviation 16 days.

(a) What percent of pregnancies last less than 240 days (that's about 8 months)?

$z = \frac{240-266}{16} = -1.63 \rightarrow .0516 \quad | \quad 5.16\%$

(b) What percent of pregnancies last between 240 and 270 days (roughly b/w 8 months and 9 months)?

$z = \frac{240-266}{16} = -1.63$   
 $z = \frac{270-266}{16} = 0.25$   
 $z = 270-266 = 0.25$



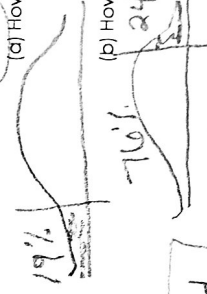
4. Rachel wonders how much television kids watch these days. She figures children television usage in a day is Normally distributed with a mean of 2 hours and a standard deviation of 0.3 hours.

(a) How many hours of television do the bottom 19% of children television use?

$-0.88 = \frac{x-2}{0.3} \quad | \quad x = 1.74 \text{ hours and up}$

(b) How many hours of television do the top 24% of children television use?

$0.71 = \frac{x-2}{0.3} \quad | \quad x = 2.21 \text{ hours and up}$



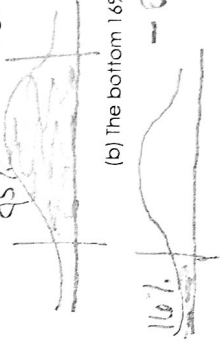
The annual rate of return on stock indexes (which combine many individual stocks) is very roughly Normal. Since 1945, the Standard & Poor's 500 index has had a mean yearly return of 12%. With a standard deviation of 1.65%. Take this Normal distribution to be the distribution of yearly returns over a long period.

(a) In what range do the middle 95% of all yearly returns lie?

$-21 \text{ to } 45$   
 (a o up/down)

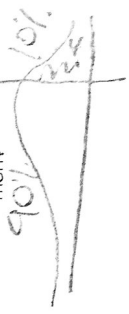
(b) The bottom 16% of all yearly returns will be lower than what percent?

$-0.97 = \frac{x-12}{1.65} \quad | \quad x = -4.34$



6. The distribution of heights of adult American men is approximately normal with mean 69 inches and standard deviation 2.5 inches. How tall must a man be in the tallest 10% of adult men?

$1.28 = \frac{x-69}{2.5}$



$x = 72.2 \text{ inches or more}$

The demand for meat at a grocery store during any week is approximately normally distributed with a mean demand of 5000 pounds and a standard deviation of 300 pounds.

(a) If the store has 5300 pounds of meat, what is the probability that it is overstocked?

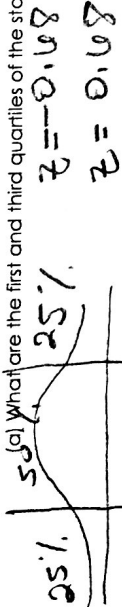
$$P(X > 5300) \quad z = \frac{5300 - 5000}{300} = 1 \quad | -0.8413 = 0.1587$$

15.87%

(b) How much meat should the store have in stock per week so as not to run short more than 10 percent of the time?

8. The median of any normal distribution is the same as its mean. We can use the standard normal table to find the quantiles and related descriptive measures for normal distributions.

(a) What are the first and third quartiles of the standard normal distribution?



$$z = -0.675$$

$$z = 0.675$$

(b) What is the value of the IQR for the standard normal distribution?

$$0.675 - (-0.675) = 1.35$$

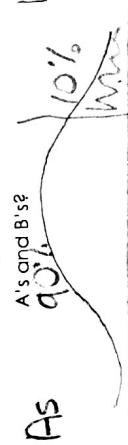
(c) What percent of the observations in the standard normal distribution are suspected outliers according to the 1.5 x IQR criterion?

$$Q3 + 1.5(1.35) = 2.72$$

$$Q1 - 1.5(1.35) = -2.72$$

1.66%

9. Scores of students on a test are approximately normally distributed with a mean score of 70 points and a standard deviation of 10 points. It is decided to give A's to 10 percent of the students and B's to 23 percent of the students. What scores should be assigned to A's and B's?



$$1.28 = \frac{x - 70}{10}$$

$$x = 82.8 \text{ and up}$$

$$0.44 = \frac{x - 70}{10} = 74.4$$

74.4 to 82.8 get A's

10. The diameter of a lead shot has a normal distribution with a mean diameter equal to 2 inches and a standard deviation equal to 0.05 inches. Find what diameter a circular hole should be so that only 3 percent of the lead shots can pass through it.



$$-1.88 = \frac{x - 2}{0.05}$$

1.904 inches