Finding the Roots of a Polynomial Given a Clue

Not all polynomial functions have an obvious factor pattern. It is important for us to be able to find the roots of the function using a combination of methods.

Step 1: Use synthetic division and the given root to find the depressed row of the polynomial.

Step 2: See if the depressed polynomial will factor, if not- use quadratic formula or square root property as the problem dictates.

Step 3: Remember to check the degree of the polynomial and account for all the solutions. _____

EXAMPLE #1 Find the roots of the polynomial $f(x) = x^3 - x^2 - 11x + 3$ given that $f(-3) = 0$

$$\begin{array}{r|cccc} -3 & 1 & -1 & -11 & 3 \\ & \downarrow & -3 & 12 & -3 \\ \hline & 1 & -4 & 1 & 0 \end{array}$$

$x^2 - 4x + 1$

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$a = 1 \quad b = -4 \quad c = 1$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{4 \pm \sqrt{(-4)^2 - 4(1)(1)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{12}}{2} = \frac{4 \pm 2\sqrt{3}}{2}$$

$$= 2 \pm \sqrt{3}$$

$2 \pm \sqrt{3}, -3$

zeros

$2 + \sqrt{3}, 2 - \sqrt{3}, -3$

~~Factors~~ $(x - (2 + \sqrt{3}))$

EXAMPLE #2 Find all the roots of the polynomial $f(x) = x^4 - 9x^3 + 23x^2 - 81x + 126$ given that $x = 2$ and $x = 7$ are roots of the function.

$$
\begin{array}{c|cccc|c}
2 & 1 & -9 & 23 & -81 & 126 \\
 & \downarrow & 2 & -14 & 18 & -126 \\
\hline
 & 1x^3 & -7x^2 & 9x & -63 & 0
\end{array}
$$

$$1x^3 - 7x^2 + 9x - 63 = 0$$

$$\begin{array}{c|cccc} 7 & 1 & -7 & 9 & -63 \\ & \downarrow & 7 & 0 & 63 \\ \hline & 1x^2 & 0x & 9 & 0 \end{array}$$

$x^2 + 9 = 0$

$-9 \quad -9$

$\sqrt{x^2} = \sqrt{-9}$

$x = \pm 3i$

Zeros: $7, 2, 3i, -3i$

EXAMPLE #3 Find all the solutions of the equation $y = x^3 + 3x^2 - 16x - 48$ given that (x+3) is a factor.

$$\begin{array}{r|cccc} -3 & 1 & 3 & -16 & -48 \\ & \downarrow & -3 & 0 & 48 \\ \hline & 1 & 0 & -16 & 0 \end{array}$$

$x^2 - 16 = 0$

$+16 \quad +16$

$\sqrt{x^2} = \sqrt{16}$

$x = \pm 4, -3$