Inverses of Functions

Domain of a Function:
(left, right) x-values

Range of a Function:
(low, high) y-values

Inverse of a Function:
switch the x and y values, then solve for y.

Steps to find the inverse of a function:

1. Switch the x and y variables.
2. Solve for y.

Examples: Find the inverses of the following functions.

1. $\cancel{f(x)}\ y = x + 4$

$x = y + \cancel{4}$
$-4 \quad\quad -\cancel{4}$

$\boxed{y = x - 4}$

2. $y = -12x + 4$

$x = -12y + \cancel{4}$
$-4 \quad\quad -\cancel{4}$

$\frac{x-4}{-12} = \frac{-\cancel{12}y}{-\cancel{12}}$

$\boxed{y = \frac{x-4}{-12}}$

3. $f(x) = x^2 + 2$

$x = y^2 + \cancel{2}$
$-2 \quad\quad -\cancel{2}$

$\sqrt{x-2} = \sqrt{y^2}$

$\boxed{y = \sqrt{x-2}}$

4. $y = \sqrt{2x + 5}$

$x^2 = \sqrt{2y+5}^{\,2}$

$x^2 = 2y + 5$
$-5 \quad\quad -5$

$\frac{x^2 - 5}{2} = \frac{\cancel{2}y}{\cancel{2}}$

$\boxed{y = \frac{x^2-5}{2}}$

5. $\{(-3,6), (0,1), (-9,-3), (6,8), (-4,-4)\}$

$(6,-3)\ (1,0)\ (-3,-9)\ (8,6)\ (-4,-4)$

Verify that the functions are inverses of each other

1. $f(x) = x + 4$ $\quad g(x) = x - 4$

$f(g(x)) = f(x-4)$

$= (x-\cancel{4}) + \cancel{4}$

$= \boxed{x}$

$g(f(x)) = g(x+4)$

$= (x+\cancel{4}) - \cancel{4}$ Yes

$= \boxed{x}$

2. $f(x) = -12x + 4$ $\quad g(x) = \dfrac{x-4}{-12}$

$f(g(x)) = f\left(\frac{x-4}{-12}\right)$

$= \frac{-\cancel{12}}{1}\left(\frac{x-4}{\cancel{-12}}\right) + 4$

$= x - \cancel{4} + \cancel{4} = \boxed{x}$

$g(f(x)) = g(-12x+4)$

$= \frac{(-12x\cancel{+4}) \cancel{-4}}{-12} = \frac{-12x}{-12}$

$\boxed{\text{Yes}}$

$= \boxed{x}$

3. $f(x) = x^2 + 2$ $\quad g(x) = \pm\sqrt{x-2}$

$f(g(x)) = f(\sqrt{x-2})$

$= \sqrt{x-2}^{\,2} + 2$

$= x - 2 + 2 = \boxed{x}$

$g(f(x)) = g(x^2+2)$

$= \sqrt{(x^2\cancel{+2})\cancel{-2}}$ $\quad \boxed{\text{Yes}}$

$= \sqrt{x^2} = \boxed{x}$

4. $f(x) = \sqrt{2x + 5}$ $\quad g(x) = \dfrac{x^2 - 5}{2}$