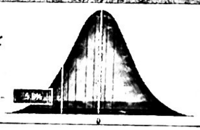
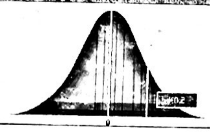
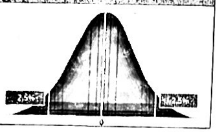
**Statistical Reasoning**
**Hypothesis Tes**

Name: ______ Date: ______ Class: ______

**Hypothesis Testing about the Mean**

**State:** Hypothesis and data

- Hypothesis: Null hypothesis ($H_0$) and Alternative hypothesis ($H_a$)
- Statistical Data
    - Parameter: Population mean (μ) or proportion (p)
    - Standard deviation: Population std. dev. (σ) or sample std. dev. (s)
    - Significance level (α): the critical value where the null hypothesis may be rejected.
        - 10% (0.1), 5% (0.05), or 1% (0.01)

| Decreased: Lower-tailed test | Increased: Upper-tailed test | Changed: Two-tailed test |
| --- | --- | --- |
| $H_0: \mu = x$<br>$H_a: \mu < x$  | $H_0: \mu = x$<br>$H_a: \mu > x$  | $H_0: \mu = x$<br>$H_a: \mu \neq x$  |

**Plan:** Select Appropriate Test Type

- Test statistic: formula and substitution—identify the appropriate test and substitute

| Z-Test | T-Test | 1 Proportion |
| --- | --- | --- |
| *If population mean and population standard deviation is known (σ)* | *If population mean and sample standard deviation is known (s)* | *If population proportion sample proportion is known* |
| $z-score = \frac{\bar{x}-\mu}{\frac{\sigma}{\sqrt{n}}}$ | $z-score = \frac{\bar{x}-\mu}{\frac{s}{\sqrt{n}}}$ | $z-score = \frac{\hat{p}-p}{\sqrt{\frac{p(1-p)}{n}}}$ |

**Do:** Compute test statistic (p-value)

**Conclude:** Reject Ho or Fail to Reject Ho

- Reject Ho: (is p-value < alpha?)—a small p-value provides evidence against the null hypothesis, because data has been observed that would be unlikely if the null hypothesis were correct.

**"With a p value of ____ at the ____ % significance level we have sufficient evidence to reject the null hypothesis and can conclude** (*insert alternative hypothesis in context here*)**"**

- Fail to Reject Ho: (is p-value > alpha?)— a large p-value does not provide enough evidence to reject the null in favor of the alternative.

**"With a p value of ____ at the ____ % significance level we don't have sufficient evidence to reject the null hypothesis and cannot conclude** (*insert alternative hypothesis in context here*)**"**



**Example:**

The National Center for Health Statistics (NCHS) published a report in 2005 which indicated that in 2002 Americans paid an average of $3,302 per year on health care and prescription drugs. An investigator hypothesizes that in 2005 expenditures have decreased primarily due to the availability of generic drugs. To test the hypothesis, a sample of 100 Americans were selected and their expenditures on health care and prescription drugs in 2005 were found to have a sample mean of $3,190. Is there statistical evidence of a reduction in expenditures on health care and prescription drugs in 2005 with a 5% level of significance?

S. dev. = $600

- **State.** Set up hypotheses and identify statistical data

Ho μ = $3,302
Ha μ < $3,302

μ = $3,302
n = 100
$\bar{x}$ = $3,190
α = .05

- **Plan.** Select the appropriate and consider conditions

- left-tailed (<)
- z-test
Find p-value, reject Ho if p<.05.

- **Do.** Compute the statistic.

Stat
~~and~~ Tests
Z-Test

p-value
p= 0.03097

- **Conclude:**

P-value = p=0.03 z-score = z=-1.87

Reject the null / Fail to Reject the null

b/c .03 < .05

With a p value of .03 at the 5 % significance level we (don't have/have) sufficient evidence to reject the null hypothesis and (can/cannot) conclude that the mean health care expenses are less than $3,302.
(*insert alternative hypothesis in context here*)

8-3 Hypothesis Testing (z Test)

The z test: a statistical test for the mean of a population. It can be used when $n \geq 30$, or when the population is normally distributed and $\sigma$ is known.

Steps for solving *z test*:

1. State the hypotheses and identify the claim
2. Find the critical value(s) – $z^*$.
3. Compute the test value.
4. Make the decision to reject or not reject the null hypothesis (If test value is inside the critical region – reject null hypothesis. If test value is inside non-critical – cannont reject null hypothesis.)
5. Summarize the results.

Calc. $\{$ 2., 3.

Ex: A researcher reports that the average salary of assistant professors is more than $42,000. A sample of 30 assistant professors has a mean salary of $43,260. At $\alpha = 0.05$, test the claim that assistant professors earn more than $42,000 a year. The standard deviation of the population is $5,230.

Ho $\mu = \$42{,}000$ $\mu = \$42{,}000$ $\bar{x} = \$43{,}260$
Ha $\mu > \$42{,}000$ $n = 30$ $\alpha = .05$ $\sigma = \$5{,}230$

P-value = .093
Fail to Reject Ho

There is not enough evidence to conclude that the mean salary is greater than $42,000.

Ex: A researcher claims that the average cost of men's athletic shoes is less than $80. He selects a random sample of 36 pairs of shoes from a catalog and finds the following costs (in dollars). Is there enough evidence to support the researcher's claim at $\alpha = 0.10$?

$\bar{X} = 75$
$S_x = 19.2$

60 70 75 55 80 55 50 40 80 70 50 95 120 90 75 85 80
60 110 65 80 85 85 45 75 60 90 90 60 95 110 85 45 90
70 70

Ho $\mu = \$80$ $n = 36$ t-test (no $\sigma$)
Ha $\mu < \$80$ $\alpha = 0.10$

$p = .06$ $\alpha = 0.10$ Reject Ho

There is enough evidence to reject Ho and we can conclude that the mean cost of shoes is less than $80.

Ex: The medical rehabilitation Education Foundation reports that the average cost of rehabilitation for stroke victims is $24,672. To see if the average cost of rehabilitation is different at a particular hospital, a researcher selects a random sample of 35 stroke victims at the hospital and finds that the average cost of their rehabilitation is $25,226. The standard deviation of the population is $3251. At $\alpha = 0.01$, can it be concluded that the average cost of stroke rehabilitation at a particular hospital is different from $24,672?

Ho $\mu = \$24{,}672$ $\mu = \$24{,}672$ $\sigma = \$3251$
Ha $\mu \neq \$24{,}672$ $n = 35$ $\alpha = .01$
$\bar{x} = \$25{,}226$

$p = 0.31$
Fail to Reject Ho

There is not enough evidence to conclude that the avg. cost is different from $24,672

P-value: the probability of getting a sample statistic (such as the mean) or a more extreme sample statistic in the direction of the alternative hypothesis when the null hypothesis is true.

Steps for *P-value*:

1. State the hypotheses and identify the claim.
2. Compute the test value.
3. Find the P-value.
4. Make the decision.
5. Summarize the results.

Ex: A researcher wishes to test the claim that the average age of lifeguards in Ocean City is greater than 24 years. She selects a sample of 36 guards and finds the mean of the sample to be 24.7 years, with a standard deviation of 2 years. Is there evidence to support the claim at $\alpha = 0.05$? Use the P-value method.