Statistical Reasoning
Hypothesis Tests

Name: ______ Date: ______ Class: ______

Hypothesis Testing of the Mean Practice

1. The college bookstore tells prospective students that the average cost of its textbooks is $52 with a standard deviation of $4.50. A group of smart statistics students thinks that the average cost is higher. In order to test the bookstore's claim against their alternative, the students will select a random sample of size 100. Assume that the mean from their random sample is $52.80. Perform a hypothesis test (4 step procedure outlined in class) at the 5% level of significance and state your conclusion.

- **State:** H₀: μ_{cost} = $52.
  Hₐ: μ_{cost} > $52.

- **Plan:** $n=100$ **Test type:** right tailed ($\alpha=.05$)
  $\bar{x}=52.80$
- **Do:** $\sigma=4.50/\sqrt{100}$ **P-value:** $1-.9623=0.0377$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.0377 at the 5 % significance level we **(don't have/have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the average cost of textbooks is greater than $52.
(*insert alternative hypothesis in context here*)

2. A certain chemical pollutant in the Genesee River has been constant for several years with mean μ = 34 ppm (parts per million) and standard deviation σ = 8 ppm. A group of factory representatives whose companies discharge liquids into the river is now claiming that they hav lowered the average with improved filtration devices. A group of environmentalists will test to see if this is true at the 1% level of significance. Assume that their sample of size 50 gives a mea of 32.5 ppm. Perform a hypothesis test at the 1% level of significance and state your decision.

- **State:** H₀: $\mu = 34$ ppm
  Hₐ: $\mu < 34$ ppm
- **Plan:** $z=\frac{32.5-34}{8/\sqrt{50}}$ **Test type:** left tailed ($\alpha=.01$) $1\%=.01$
- **Do:** **P-value:** 0.0924
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.0924 at the 1 % significance level we **(don't have/ have)** sufficien evidence to reject the null hypothesis and **(can/cannot)** conclude that the avg pollutant level is less than 34 ppm.
(*insert alternative hypothesis in context here*)

$\sigma \to$ z-test $\qquad$ S = t-test

3. A manufacturing process produces ball bearings with diameters that have a normal distribu with known standard deviation of .04 centimeters. Ball bearings with diameters that are too small or too large are undesirable. To test the claim that μ= 0.50 centimeters, perform a two-tailed hypothesis test at the 10% level of significance. Assume that a random sample of 25 gave a mean diameter of 0.51 centimeters. Perform a hypothesis test and state your decision.

- **State:** **H₀:** $\mu = .50$ cm.

  Hₐ: $\mu \neq .50$ cm

(Calc p-value 0.2112)

- **Plan:** $z = \frac{.51 - .50}{\frac{.04}{\sqrt{25}}}$ **Test type:** Two tailed (α = .10)
- **Do:** **P-value:** $(1 - .8943) \times 2 = 0.2114$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of .2112 at the 10 % significance level we **(don't have/ have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the mean diameter is not equal to .50 cm.

(insert alternative hypothesis in context here)

4. Here are measurements (in millimeters) of a critical dimension for 16 auto engine crankshafts:

224.120	224.001	224.017	223.982	223.989	223.961
223.960	224.089	223.987	223.976	223.902	223.980
	224.098	224.057	223.913	223.999	

The mean dimension is supposed to be 224 mm, and the variability of the manufacturing process is known to be around 0.004. Using an α of .05, is there evidence that the mean dimension of these measurements is $\boxed{\text{greater than}}$ 224 mm?

- **State:** **H₀:** $\mu = 224$mm

  Hₐ: $\mu > 224$mm

z-test

$\mu = 224$mm

$\sigma = 0.004$

$\alpha = .05$

- **Plan:** **Test type:** Right tailed z-test
- **Do:** **P-value:** $p = 0.026$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.026 at the 5 % significance level we **(don't have/ have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude the mean dimension is greater than 224mm.

(insert alternative hypothesis in context here)

5. The level of calcium in the blood of healthy young adults varies with a mean of about 9.5 milligrams per deciliter with standard deviation 0.41. A clinic in a rural area measured the blood calcium level of 180 healthy pregnant women during their first trimester of pregnancy. If the mean of this sample was 9.57 mg, 9.579is this an indication that the mean calcium level in the population of pregnant rural women differs from 9.5? Use a hypothesis test and a 1% significance level to form your conclusion.

- **State:** **H₀:** $\mu = 9.5$ mg.

  Hₐ: $\mu \neq 9.5$ mg.

calc $p = 0.023$

- **Plan:** **Test type:** Two-tailed ($\alpha = .01$)
- **Do:** $z = \dfrac{9.57 - 9.5}{\frac{0.41}{\sqrt{180}}}$ **P-value:** $(1 - .9890) \times 2 = 0.022$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.022 at the .01 % significance level we **(don't have/ have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the mean calcium level is different from 9.5 mg.

(insert alternative hypothesis in context here)

6. The mean yield of corn in the United States is about 120 bushels per acre. A survey of a SRS of 50 farmers selected from the population of all commercial corn growers this year gives a sample mean yield of 123.6 bushels per acre. Is there evidence at the .05 level that the population mean this year is more than 120 bushels/acre? The standard deviation of corn yield has historically been σ = 10 bushels per acre.

- **State:** **H₀:** $\mu = 120$ bushels.

  Hₐ: $\mu > 120$ bushels.

$\alpha = .05$

- **Plan:** **Test type:** Right Tailed ($\alpha = .05$)
- **Do:** $z = \dfrac{123.6 - 120}{\frac{10}{\sqrt{50}}}$ **P-value:** $1 - 0.9945 = 0.0054$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.0054 at the 5 % significance level we **(don't have/ have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the avg yield of corn is more than 120 bushels/acre.

(insert alternative hypothesis in context here)

7. Sweet corn of a certain variety is known to produce individual ears of corn with a mean weight of 8 ounces and a standard deviation of 0.8 ounces. A farmer is testing a new fertilizer designed to produce larger ears of corn, as measured by their weight. He finds that 28 randomly-selected ears of corn grown with this fertilizer have a mean weight of 8.25 ounces. There are no outliers in the data. Do these samples provide convincing evidence at the $\alpha = 0.05$ level that the fertilizer had a positive impact on the weight of the corn ears?

- **State:** H_o: $\mu = 8$ oz.
  H_a: $\mu > 8$ oz.
- **Plan:** **Test type:** Right tailed
- **Do:** **P-value:** 0.049 $\alpha = 0.05$
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.049 at the 0.05 % significance level we **(don't have/have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the avg weight of corn is greater than 8oz.

(insert alternative hypothesis in context here)

8. Is Times New Roman the easiest font to read? Adults can read four paragraphs of text in the common Times New Roman font in an average time of 22 seconds with a standard deviation of 14 seconds. Researchers asked a random sample of 24 adults to read this text in the ornate font named Gigi. Here are their times in seconds:

23.2 21.2 28.9 27.7 29.1 27.3 16.1 22.6 25.6 34.2 23.9 26.8
20.5 34.3 21.4 32.6 26.2 34.1 31.5 24.6 23.0 28.6 24.4 28.1

Do these data provide good evidence at the 10% significance level that the mean reading time for Gigi differs from the Times New Roman average? Carry out an appropriate test to help you answer this question.

- **State:** H_o: $\mu = 22$ seconds.
  H_a: $\mu \neq 22$ seconds.

$\mu = 22$ sec $n = 24$
$\sigma = 14$ sec $\alpha = .10$

- **Plan:** **Test type:** Two-tailed z-test
- **Do:** **P-value:** 0.116
- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.116 at the 10 % significance level we **(don't have/have)** sufficient evidence to reject the null hypothesis and **(can/cannot)** conclude that the average reading time is different from 22 sec.

(insert alternative hypothesis in context here)