Warm Up

Simplify each expression.

1) $\dfrac{2}{m-2} + \dfrac{-5}{m+3}$

LCD $(m-2)(m+3)$

$\dfrac{(m+3)}{(m+3)}\dfrac{2}{m-2} + \dfrac{-5}{m+3}\dfrac{(m-2)}{(m-2)}$

$\dfrac{2m+6}{(m+3)(m-2)} + \dfrac{-5m+10}{(m+3)(m-2)} = \dfrac{-3m+16}{(m+3)(m-2)}$

Let's remember some important facts about dealing with rational expressions!

1. When adding or subtracting fractions we must have a common denominator.

2. When multiplying fractions we can cancel if we see there is the same factor on the top and bottom.

3. When dividing fractions we must multiply and flip the second/bottom fraction.

4. Don't forget to always make sure your expressions are in standard form.

No factoring needed:

1. $\dfrac{\dfrac{3}{x}}{\dfrac{9}{x^3}}$ ← ÷

$$\frac{3}{x} \div \frac{9}{x^3} = \frac{3}{x} \cdot \frac{x^3}{9}$$

$$= \frac{3x^3}{9x^1} = \boxed{\frac{1x^2}{3}}$$

2. $\dfrac{\dfrac{2m}{n^3}}{\dfrac{8m^3}{n^2}} = \dfrac{2m}{n^3} \div \dfrac{8m^3}{n^2}$

$$= \frac{2m}{n^3} \cdot \frac{n^2}{8m^3}$$

$$= \frac{2m^1n^2}{8m^3n^3} = \boxed{\frac{1}{4m^2n}}$$

Complex Fractions Involving Factoring

1. $\dfrac{\dfrac{x^2-25}{9}}{\dfrac{x+5}{3}}$

$$\frac{x^2-25}{9} \div \frac{x+5}{3} = \frac{x^2-25}{9} \cdot \frac{3}{x+5}$$

$$= \frac{(x+5)(x-5)}{9} \cdot \frac{3}{x+5} = \frac{3(x-5)}{9} = \boxed{\frac{1(x-5)}{3}}$$

2. $$\frac{\dfrac{y^2+19y+84}{4y-4}}{\dfrac{y^2+9y+14}{2y-2}}$$

$84 = 7 \cdot 12$, $19 = 7 + 12$

$$\frac{y^2+19y+84}{4y-4} \cdot \frac{2y-2}{y^2+9y+14} = \frac{(y+7)(y+12)}{4(y-1)} \cdot \frac{2(y-1)}{(y+7)(y+2)}$$

GCF

GCF

$14 = 7 \cdot 2$, $9 = 7 + 2$

$$= \frac{2(y+12)}{4(y+2)} = \boxed{\frac{y+12}{2(y+2)}}$$

3. $$\frac{\frac{x^2-1}{3x+2}}{\frac{x+1}{3x^2-x-2}}$$

$(x-3)(x+2)$

x^2-x-6

$-3 \times 2 = -6$, $-3 + 2 = -1$

$$\frac{x^2-1}{3x+2} \cdot \frac{3x^2-x-2}{x+1}$$

$$\frac{(x+1)(x-1)}{3x+2} \cdot \frac{(x-1)(3x+2)}{x+1}$$

$$= \boxed{(x-1)(x-1) \text{ OR } (x-1)^2}$$

Complex Fractions Involving Addition/Subtraction

1. $$\frac{\frac{1}{x}-\frac{1}{y}}{1+\frac{1}{x}}$$

LCD xy

Top: $y\cdot\frac{1}{x}+\frac{-1\cdot x}{y\cdot x}$

$$=\frac{y}{xy}+\frac{-x}{xy}=\frac{y-x}{xy}$$

LCD x

Bottom: $x\cdot\frac{1}{x}+\frac{1}{x}$

$$=\frac{x}{x}+\frac{1}{x}=\frac{x+1}{x}$$

$$\frac{\frac{y-x}{xy}}{\frac{x+1}{x}}$$

$$\frac{y-x}{xy}\cdot\frac{x}{x+1}$$

$$\boxed{\frac{y-x}{y(x+1)}}$$

2. $$\frac{\frac{2}{x-y}+\frac{1}{x+y}}{\frac{1}{x-y}}$$

LCD $(x-y)(x+y)$

Top $\frac{2}{x-y}(x+y) + \frac{1}{x+y}(x-y)$

$$\frac{2x+2y}{(x+y)(x-y)} + \frac{x-y}{(x+y)(x-y)} = \frac{3x+y}{(x+y)(x-y)}$$

$$\frac{\frac{3x+y}{(x+y)(x-y)}}{\frac{1}{x-y}}$$

$$\frac{3x+y}{(x+y)(x-y)} \cdot \frac{x-y}{1}$$

$$= \boxed{\frac{3x+y}{x+y}}$$