How Can I Solve a Rational Equation?

Before you begin solving rational equations, it is important to understand the idea of **extraneous solutions.**

An *extraneous solution* is a solution that is derived algebraically but does not "satisfy" the original equation (i.e. its value is not in the domain). It is important to determine before you begin the solution process what the answers CANNOT be. The solutions to a rational equation are any values that would create a __zero__ in the __denominator__ when substituted in.

extraneous

Consider the following expressions and determine which values would make them *undefined*.

A. $\dfrac{6+x}{x+2}$

$x+2=0$
$-2 \quad -2$
$x=-2$
$\boxed{x \neq -2}$

B. $\dfrac{5}{\cancel{x^2-4}}$

$(x+2)(x-2)$
$\boxed{x \neq -2, 2}$

C. $\dfrac{3x}{2x+1}$

$2x+1=0$
$-1 \quad -1$
$\dfrac{2x}{2}=\dfrac{-1}{2}$
$\boxed{x \neq -1/2}$

D. $\dfrac{1}{4x}$

$\dfrac{\cancel{4}x}{\cancel{4}}=\dfrac{0}{4}$
$\boxed{x \neq 0}$

E. $\dfrac{x+3}{8x^2+16x}$

F. $\dfrac{2x}{\cancel{x^2+7x+10}}$

10
5 2
7

$(x+5)(x+2)$
$\boxed{x \neq -5, -2}$

G. $\dfrac{7}{2-x}$

$x \neq 2$

H. $\dfrac{4x}{7x-11}$

$x \neq \dfrac{11}{7}$

Procedure for Solving a Rational Equation

(excluded values)

Step 1: Factor the denominators of all rational expressions. Identify any values of the variable for which any expression is undefined.

Step 2: Identify the LCD of all terms in the equation.

Step 3: Multiply both sides of the equation by the LCD. (over 1)

Step 4: Solve the resulting equation.

Step 5: Check the potential solutions in the original equation. Note that any value from Step 1 for which the equation is undefined cannot be a solution to the equation.

Teacher worked example	You try
With one solution... $\frac{3}{4x}=\frac{5}{x+2}$ $x \neq 0, -2$ LCD: $4x(x+2)$ $\frac{\cancel{4x}\cancel{(x+2)}}{1}\cdot\frac{3}{\cancel{4x}}=\frac{5}{\cancel{x+2}}\cdot\frac{4x\cancel{(x+2)}}{1}$ $3(x+2)=20x$ $3x+6=20x$ $-3x \quad -3x$ $6=17x$ $\frac{6}{17}=\frac{17x}{17}$ $\boxed{x=6/17}$	$\frac{3}{x+2}=\frac{6}{x-1}$ $x \neq -2, 1$ LCD $(x+2)(x-1)$ $\frac{\cancel{(x+2)}(x-1)}{1}\cdot\frac{3}{\cancel{x+2}}=\frac{6}{\cancel{x-1}}\cdot\frac{(x+2)\cancel{(x-1)}}{1}$ $3(x-1)=6(x+2)$ $3x-3=6x+12$ $-3x \quad -3x$ $-3=3x+12$ $-12 \quad -12$ $-15=3x$ $\frac{-15}{3}=\frac{3x}{3}$ $\boxed{x=-5}$

With two solutions. . .

$$\frac{7}{x+3} = \frac{x}{4}$$

$x \neq -3$

LCD $4(x+3)$

$$\frac{4\cancel{(x+3)}}{1} \cdot \frac{7}{\cancel{x+3}} = \frac{x}{\cancel{4}} \cdot \frac{\cancel{4}(x+3)}{1}$$

$$28 = x(x+3)$$

$$28 = x^2 + 3x$$

$$-28 \qquad\qquad -28$$

$$0 = x^2 + 3x - 28$$

$$0 = (x+7)(x-4)$$

$x+7=0 \qquad x-4=0$

$$\boxed{x=-7 \qquad x=4}$$

-28

7 -4

3

Teacher worked example

With one solution...

$$\frac{4}{x}+\frac{5}{2}=-\frac{11}{x}$$

$x \neq 0$

LCD $2x$

$$\frac{\cancel{2x}}{1}\cdot\frac{4}{\cancel{x}}+\frac{5}{\cancel{2}}\cdot\frac{\cancel{2}x}{1}=-\frac{11}{\cancel{x}}\cdot\frac{2\cancel{x}}{1}$$

$$\cancel{8}+5x=-22$$
$$\cancel{-8}\qquad -8$$

$$\frac{\cancel{5}x}{\cancel{5}}=\frac{-30}{5}$$

$$\boxed{x=-6}$$

You try

$$\frac{3}{5}+\frac{1}{x}=\frac{2}{3}$$

$x \neq 0$

LCD : $15x$

$\cancel{30x}$ / 3

$$\frac{\cancel{15x}^{3x}}{1}\cdot\frac{3}{\cancel{5}_1}+\frac{1}{\cancel{x}}\cdot\frac{15\cancel{x}}{1}=\frac{2}{\cancel{3}_1}\cdot\frac{\cancel{15x}^{5x}}{1}$$

$$\cancel{9x}+15=10x$$
$$\cancel{-9x}\qquad -9x$$

$$\boxed{15=x}$$

$$\frac{10}{x(x-2)} + \frac{4}{x} = \frac{5}{x-2}$$

$x \neq 2, 0$

LCD $x(x-2)$

$$\frac{\cancel{x(x-2)}}{1} \cdot \frac{10}{\cancel{x(x-2)}} + \frac{4}{\cancel{x}} \cdot \frac{\cancel{x}(x-2)}{1} = \frac{5}{\cancel{x-2}} \cdot \frac{x\cancel{(x-2)}}{1}$$

$$10 + 4(x-2) = 5x$$

$$10 + 4x - 8 = 5x$$

$$4x + 2 = 5x$$

$$-4x \qquad -4x$$

$$\cancel{2 = x}$$

No Solution

$$\frac{6x}{x+4} + \frac{4}{1} = \frac{2x+2}{x-1}$$

$x \neq -4, 1$

LCD $(x+4)(x-1)$

$$\frac{\cancel{(x+4)}(x-1)}{1} \cdot \frac{6x}{\cancel{x+4}} + \frac{4}{1} \cdot \frac{(x+4)(x-1)}{1} = \frac{2x+2}{\cancel{x-1}} \cdot \frac{(x+4)\cancel{(x-1)}}{1}$$

$$6x(x-1) + 4(x+4)(x-1) = (2x+2)(x+4)$$

$$6x^2 - 6x + 4(x^2 \cancel{-x} \overset{3x}{\cancel{+4x}} - 4) = 2x^2 + 8x + 2x + 8$$

$$6x^2 - 6x + 4x^2 + 12x - 16 \quad = 2x^2 + 10x + 8$$

$$10x^2 + 6x - 16 \quad = \cancel{2x^2 + 10x + 8}$$

$$-2x^2 - 10x - 8 \quad \cancel{-2x^2 - 10x - 8}$$

$$8x^2 - 4x - 24 = 0$$

$$\frac{6}{x+2} + \frac{-20x}{\cancel{x^2-x-6}} = \frac{x}{x+2}$$

$(x-3)(x+2)$

$x \neq -2, 3$

LCD $(x-3)(x+2)$

$$\frac{(x-3)\cancel{(x+2)}}{1} \cdot \frac{6}{\cancel{x+2}} + \frac{-20x}{\cancel{(x-3)(x+2)}} \cdot \frac{\cancel{(x-3)(x+2)}}{1} = \frac{x}{\cancel{x+2}} \cdot \frac{(x-3)\cancel{(x+2)}}{1}$$

$$6(x-3) + -20x = x(x-3)$$

$$6x - 18 - 20x = x^2 - 3x$$

$$\cancel{-14x - 18} = x^2 - 3x$$

$$\cancel{+14x} + 18 \qquad +14x + 18$$

$$0 = x^2 + 11x + 18$$

$$0 = (x+9)(x+2)$$

$\boxed{x = -9}$ $\cancel{x = -2}$

$$\frac{36}{x^2 - 9} = \frac{2x}{x + 3} - 1$$