

Hypothesis Testing with Proportions

State: State null and alternative hypotheses and identify statistical data.

- Hypothesis: Null hypothesis (H_0) and Alternative hypothesis (H_a)
- Statistical Data
 - Parameter: Population mean (μ) or proportion (p)
 - Standard deviation: Population std. dev. (σ) or sample std. dev. (s)
 - Significance level (α): the critical value where the null hypothesis may be rejected.
 - 10% (0.1), 5% (0.05), or 1% (0.01)
 - P-hat = \hat{p} the sample proportion a result as extreme or more extreme than the one



Plan: Select the appropriate test and consider conditions.

Z-Test	T-Test	1-Proportion
If population mean and population standard deviation is known (σ)	If population mean and sample standard deviation is known (s)	If population proportion sample proportion is known
z-score = $\frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	z-score = $\frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	z-score = $\frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$

Do: Compute the p-value, show all calculations and calculator inputs

Conclude: Explain your findings in context to the problem.

- Reject H_0 : (is p-value < alpha?) — a small p-value provides evidence against the null hypothesis, because data has been observed that would be unlikely if the null hypothesis were correct.
- With a p value of _____ at the _____ % significance level we have sufficient evidence to reject the null hypothesis and can conclude (insert alternative hypothesis in context here)
- Failed to reject H_0 : (is p-value > alpha?) — a large p-value does not provide enough evidence to reject the null hypothesis in favor of the alternative.
- With a p value of _____ at the _____ % significance level we don't have sufficient evidence to reject the null hypothesis and cannot conclude (insert alternative hypothesis in context here)

Example 1: Mr. Hybart claims to produce random numbers from 1 to 6 with his "magical dice" for a particular game, but you've been keeping track. In the past 80 rolls, the number 1 has come up 3 times. You suspect that the die is producing too many 1s than it should or that he is cheating. Is there statistical evidence that he is cheating given a level of significance of 5%?

State: Set up hypotheses and identify statistical data

$H_0: P = \frac{1}{6}$
 $H_a: P \neq \frac{1}{6}$
 $P = \frac{1}{6}$ $X = 3$ $n = 80$ $\alpha = 0.05$
 Plan: Select the appropriate and consider conditions. Test type: Two tailed P Test
 Do: Compute the statistic. P-value = 0.002 z-score = -3.1

Conclude: reject the null Fail to reject the null

With a p value of 0.002 at the 5% significance level we (don't have / have) sufficient evidence to reject the null hypothesis and (can / cannot) conclude that the probability is not equal to 1/6. (insert alternative hypothesis in context here)

Example 2: An airline claims that, on average, 5% of its flights are delayed each day. On a given day, of 500 flights, 12% are delayed. Test the hypothesis that the average proportion of delayed flights is more than 5% at the 0.01 level

State: Set up hypotheses and identify statistical data

$H_0: P = 5\%$ of flights delayed
 $H_a: P > 5\%$ of flights delayed
 $P_0 = 5\%$ $X = 60$ $n = 500$ $\alpha = 0.01$
 Plan: Select the appropriate and consider conditions. Test type: Right tail p-test
 Do: Compute the statistic. P-value = 0 z-score = 7.18

Conclude: reject the null Fail to reject the null

With a p value of 0 at the 1% significance level we (don't have / have) sufficient evidence to reject the null hypothesis and (can / cannot) conclude that the percent of delayed flights is more than 5%. (insert alternative hypothesis in context here)

Example 3:

Mr. Hybart heard that one out of every 3 senior students have a job, but you believe that there's no way that the figure can be right. In one of your classes you found that much fewer of your peers had jobs than what he claimed. In a survey you administered to another class, you found that there were much more of your peers that had jobs than what Mr. H claimed. Below is the data that you collected. Use a hypothesis test to collect evidence against Mr. H's purported claim.

Class 1: Do you have a job?	
Response	Frequency
Yes	5
NO	26
Total = 31	

Sample Proportion of students with jobs
 $\frac{5 + 12}{31 + 32} = \frac{17}{63} = 0.27 = 27\%$

Class 2: Do you have a job?	
Response	Frequency
Yes	12
NO	20
Total = 32	

Mr. Hybart's claimed proportion of students with jobs
 $\frac{1}{3} = 0.333 = 33.3\%$

$H_0: P = \frac{1}{3} \quad H_a: P \neq \frac{1}{3}$

$X = 17 \quad n = 63 \quad \alpha = 0.05$

$P\text{-value} = .285$
 Fail to Reject

There is not enough evidence to reject H_0 and cannot conclude that the proportion of students with jobs is different from $\frac{1}{3}$.