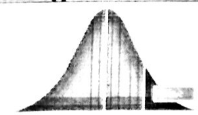
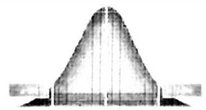
Statistical Reasoning
Hypothesis Tests

Name: ________________ Date: ________ Class: ________

Hypothesis Testing with Two Samples

***State**: State null and alternative hypotheses and identify statistical data.*

- Hypothesis: Null hypothesis (H_0) and Alternative hypothesis (H_a)
- Statistical Data
  - Parameter: Population mean (μ) or proportion (p)
  - Standard deviation: Population std. dev. (σ) or sample std. dev. (s)
  - Significance level (α): the critical value where the null hypothesis may be rejected.
    - 10% (0.1), 5% (0.05), or 1% (0.01)
  - $\bar{x}_1$ and $\bar{x}_2$ — *sample means from data sets 1 and 2*
  - s_1^2 and s_2^2 — *squared standard deviation from data sets 1 and 2 (variance)*
  - n_1 and n_1 — *sample size from data sets 1 and 2*

Decreased: Lower-tailed test	*Increased: Upper-tailed test*	*Changed: Two-tailed test*
$H_0: \mu = x$ $H_a: \mu < x$ 	$H_0: \mu = x$ $H_a: \mu > x$ 	$H_0: \mu = x$ $H_a: \mu \neq x$ 

***Plan**: Select the appropriate test and consider conditions.*

- Test statistic: formula and substitution—identify the appropriate test and substitute

Z- Test	*T- Test*	*1 Proportion*	*2 Sample*
If population mean and population standard deviation is known (σ)	*If population mean and sample standard deviation is known* (s)	*If population proportion sample proportion is known*	*If 2 samples means and 2 sample variances are known*
$z = \dfrac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}}$	$z = \dfrac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$	$z = \dfrac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{n}}}$	$t = \dfrac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$

$\bar{x}$
s

***Do**: Compute the p-value, show all calculations and calculator inputs*

***Conclude**: Explain your findings in context to the problem.*

- Reject H_0: (is p-value < alpha?)—a small p-value provides evidence against the null hypothesis, because data has been observed that would be unlikely if the null hypothesis were correct.

"With a p value of ______ at the ______ % significance level we have sufficient evidence to reject the null hypothesis and can conclude (*insert alternative hypothesis in context here*)"

- Failed to re reject H_0: (is p-value > alpha?)— a large p-value does not provide enough evidence to reject the null in favor of the alternative.

"With a p value of ______ at the ______ % significance level we don't have sufficient evidence to reject the null hypothesis and cannot conclude (*insert alternative hypothesis in context here*)"

Example 1:

In an attempt to determine if two competing brands of cold medicine contain, on the average, the same amount of acetaminophen, twelve different tablets ~~from~~ each of the two competing brands were randomly selected and tested for the amount of acetaminophen each contains. The results (in milligrams) follow. State and perform an appropriate hypothesis test. Use a significance level of 0.01. α

Brand A L1	Brand B L2
517, 495, 503, 491	493, 508, 513, 521
503, 493, 505, 495	541, 533, 500, 515
498, 481, 499, 494	536, 498, 515, 515

- **State:** Set up hypotheses and identify statistical data

H_0: $\mu_{Brand\ A} = \mu_{Brand\ B}$
H_a: $\mu_{Brand\ A} \neq \mu_{Brand\ B}$

- **Plan:** Select the appropriate and consider conditions. **Test type:** 2 sample t test
- **Do:** Compute the statistic.

P-value = 0.002 **t score =** -3.52

- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0.002 at the 1 % significance level we **(don't have/ have)** sufficient evidence to reject the null hypothesis and **(can / cannot)** conclude that the two brands have different avg. amounts of acetaminophen.
(*insert alternative hypothesis in context here*)

Example 2:
Two kinds of thread are being compared for strength. Fifty pieces of each type of thread are tested under similar conditions. The sample data on tensile strength is given in the following table. Use a hypothesis test to discover if the strength is higher in Thread B.

	Sample Size (n)	Sample mean ($\bar{x}$)	Sample Standard Deviation (s)
Thread A	50	78.3	5.62
Thread B	50	87.2	6.31

- **State:** Set up hypotheses and identify statistical data

H_0: $\mu_A = \mu_B$

H_a: $\mu_A < \mu_B$

- **Plan:** Select the appropriate and consider conditions. **Test type:** 2 sample t test

- **Do:** Compute the statistic.

P-value = 0 ~~z~~ t**-score =** −7.45

- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of 0 at the 5 % significance level we ***(don't have/ have)*** sufficient evidence to reject the null hypothesis and ***(can/cannot)*** conclude that the strength is higher in thread B.
(insert alternative hypothesis in context here)

Example 3:
A student recorded the mileage he obtained while commuting to school in his car. He kept track of the mileage for twelve different tankfuls of fuel, involving gasoline of two different octane ratings. Use a hypothesis test to investigate if the MPG is less [illegible] with the 90 Octane. His data follow:

L1 87 Octane	L2 90 Octane
26.4, 27.6, 29.7	30.5, 30.9, 29.2
28.9, 29.3, 28.8	31.7, 32.8, 29.3

- **State:** Set up hypotheses and identify statistical data

H_0: $\mu_{87} = \mu_{90}$

H_a: $\mu_{87} > \mu_{90}$

- **Plan:** Select the appropriate and consider conditions. **Test type:** 2 sample t test

- **Do:** Compute the statistic.

P-value = .99 ~~z~~ t**-score =** −3.01

- **Conclude:** **Reject the null / Fail to Reject the null**

With a p value of .99 at the 5 % significance level we ***(don't have/ have)*** sufficient evidence to reject the null hypothesis and ***(can/cannot)*** conclude that the avg mpg is less for 90 Octane.
(insert alternative hypothesis in context here)