

Solving Exponential and Logarithmic Equations 2

Exponential equations are equations in which variable expressions occur as exponents.

Logarithmic equations are equations that involve logarithms of variable expressions.

Property of Equality for Logarithmic Equations

If b , x , and y are positive numbers with $b \neq 1$, then $\log_b x = \log_b y$ if and only if $x = y$.

Ex. 1 Solve a logarithmic equation

Solve: $\log_5(6x - 16) = \log_5(x - 1)$

$$\begin{array}{r} 6x - 16 = x - 1 \\ -x \quad -x \\ \hline 5x - 16 = -1 \\ +16 \quad +16 \\ \hline 5x = 15 \\ \frac{5x}{5} = \frac{15}{5} \\ \boxed{x = 3} \end{array}$$

Solve: $\log(11) = \log(x^2 + 2)$

$$\begin{array}{r} 11 = x^2 + 2 \\ -2 \quad -2 \\ \hline 9 = x^2 \\ \sqrt{9} = \sqrt{x^2} \\ \boxed{x = \pm 3} \end{array}$$

YOU TRY!

Solve: $\ln(7x - 13) = \ln(2x + 17)$

$$\begin{array}{r} 7x - 13 = 2x + 17 \\ -2x \quad -2x \\ \hline 5x - 13 = 17 \\ +13 \quad +13 \\ \hline 5x = 30 \\ \frac{5x}{5} = \frac{30}{5} \\ \boxed{x = 6} \end{array}$$

Solve: $\log_8(x + 6) = \log_8(4 - x)$

$$\begin{array}{r} x + 6 = 4 - x \\ +x \quad +x \\ \hline 2x + 6 = 4 \\ -6 \quad -6 \\ \hline 2x = -2 \\ \frac{2x}{2} = \frac{-2}{2} \\ \boxed{x = -1} \end{array}$$

Identity property of Logarithms

If $b \neq 0$ and $\log_a b = c$, then $a^c = b$

Ex. 2 Rewrite the logarithmic function as an exponential function to solve the equation.

Solve: $\log_5(3x - 8) = 2$

$$\begin{array}{r} 5^2 = 3x - 8 \\ 25 = 3x - 8 \\ +8 \quad +8 \\ \hline 33 = 3x \\ \frac{33}{3} = \frac{3x}{3} \\ \boxed{x = 11} \end{array}$$

Solve: $\log_2(2x + 5) = 3$

$$\begin{array}{r} 2^3 = 2x + 5 \\ 8 = 2x + 5 \\ -5 \quad -5 \\ \hline 3 = 2x \\ \frac{3}{2} = \frac{2x}{2} \\ \boxed{x = 3/2} \end{array}$$

YOU TRY!

Solve: $\log_3(2x + 9) = 3$

$$\begin{array}{r} 3^3 = 2x + 9 \\ 27 = 2x + 9 \\ -9 \quad -9 \\ \hline 18 = 2x \\ \frac{18}{2} = \frac{2x}{2} \\ \boxed{x = 9} \end{array}$$

Solve: $\log_4(10x + 624) = 5$